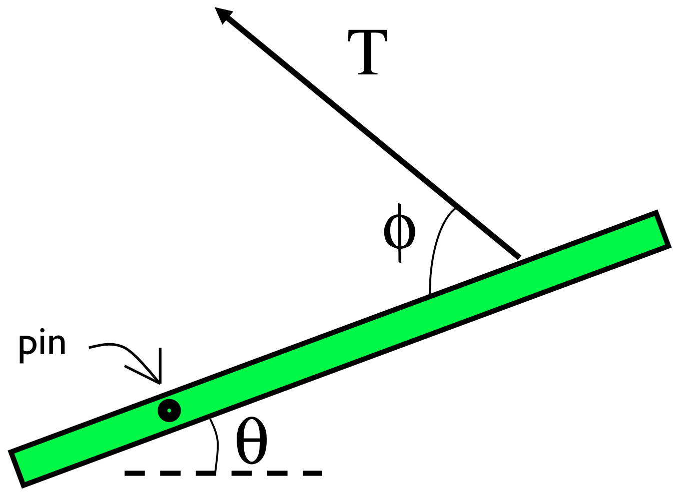
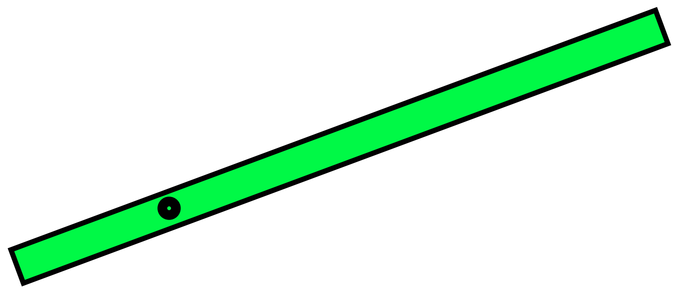
8.) A beam of length "L" is pinned at an angle θ a quarter of the way up the beam (i.e., at L/4). Tension in a rope three-quarters of the way from the end keeps it stationary. What is known is:

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

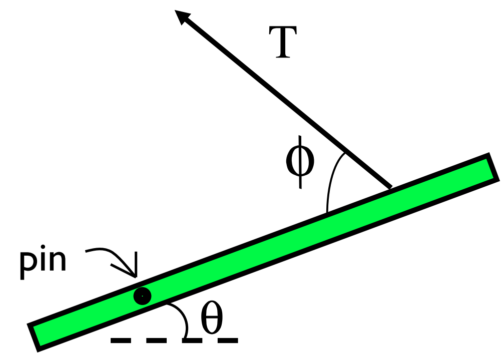


a.) Draw a f.b.d. identifying all the forces acting on the beam.



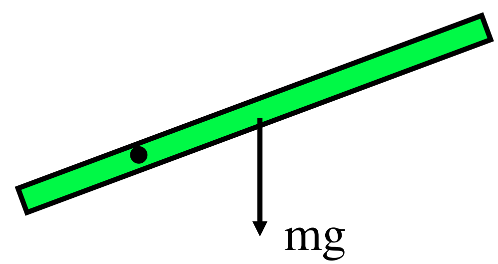
b.) What must the tension in the rope be for equilibrium?

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$



c.) Use the *Parallel Axis Theorem* to derive an expression for the beam' s *moment of inertia* about the pin.

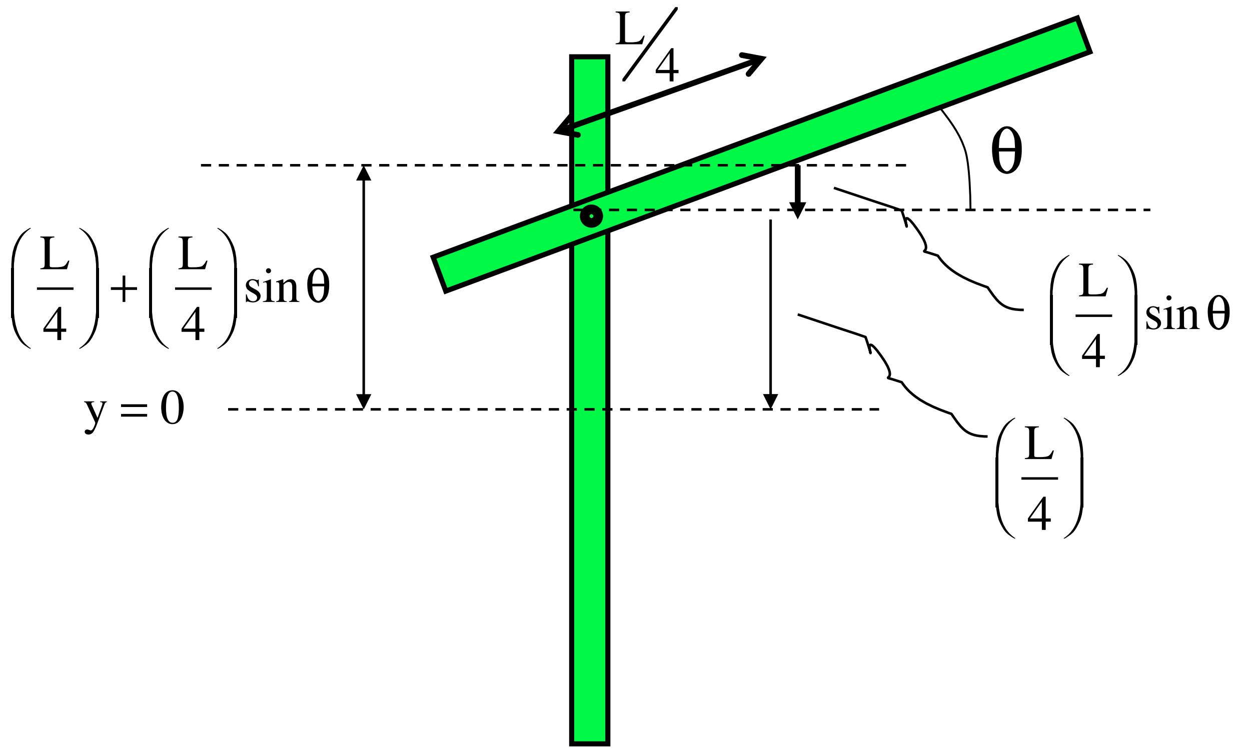
d.) The rope is cut and the beam begins to angular accelerate downward. Derive an expression for the beam's initial *angular acceleration* about the pin?



$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

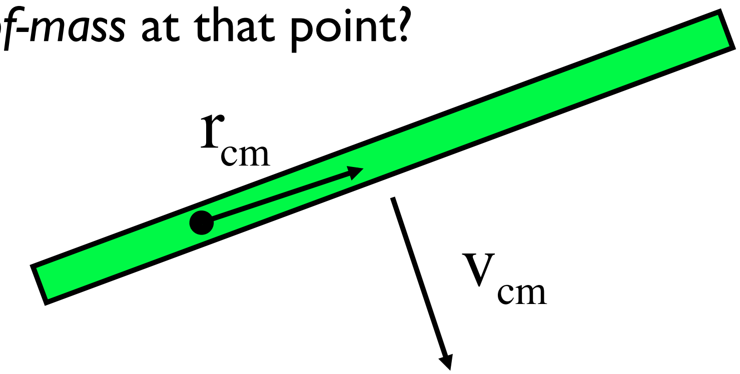
e.) What is the initial acceleration of the beam's *center of mass*?

f.) The beam rotates downward. What is its angular velocity as it passes through the vertical? (Note that I've done some geometry for you—this probably won't be the case on the test.)



$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

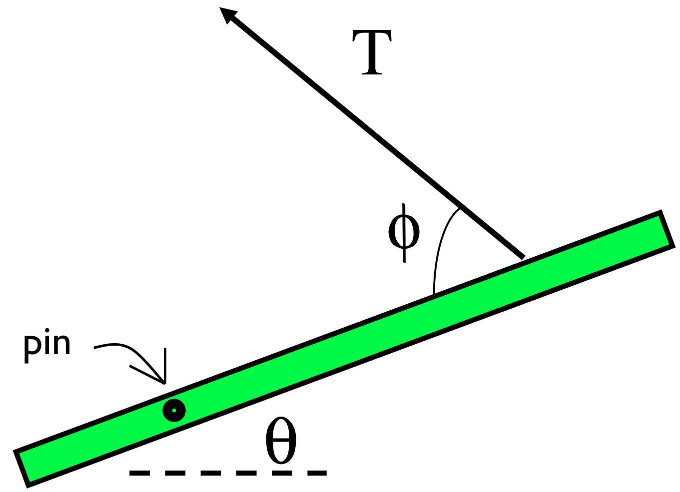
g.) What is the velocity magnitude of the beam's *center-of-mass* at that point?



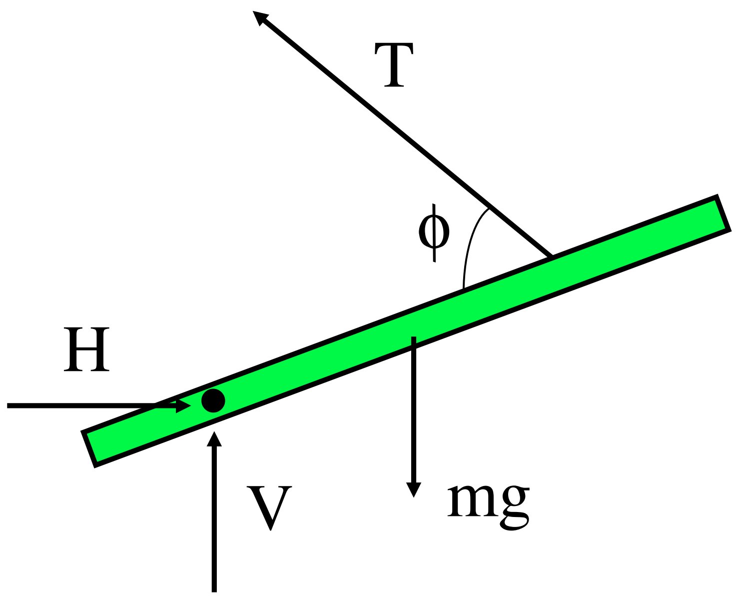
h.) What is the beam's *angular momentum* "L" about the pin at that point?

8.) A beam of length "L" is pinned at an angle θ a quarter of the way up the beam (i.e., at L/4). Tension in a rope three-quarters of the way from the end keeps it stationary. What is known is:

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

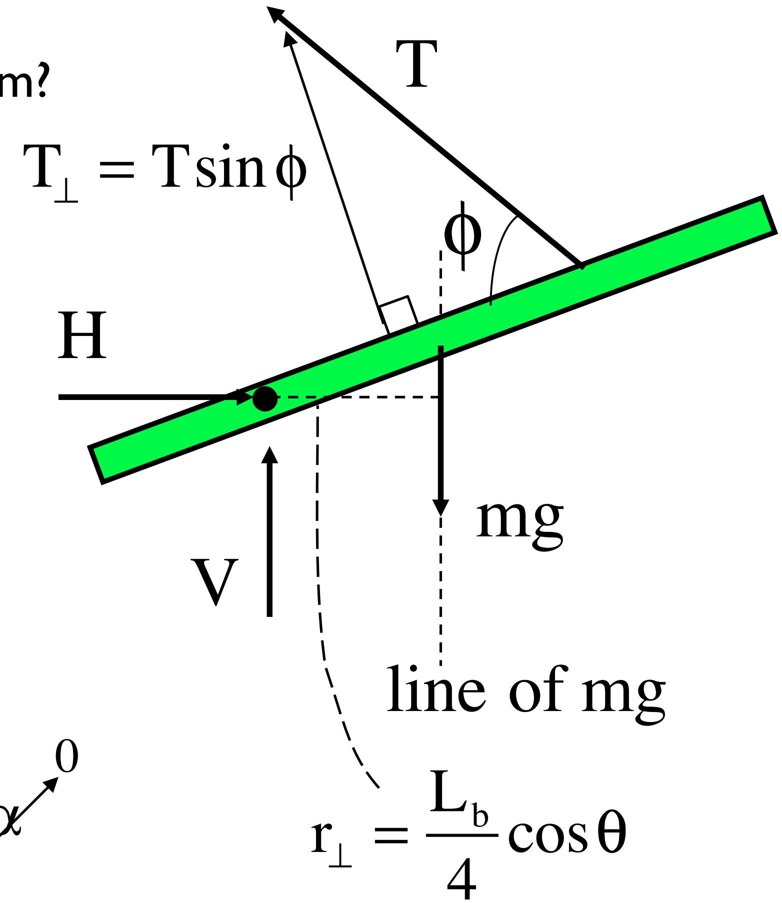


a.) Draw a f.b.d. identifying all the forces acting on the beam.



$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

b.) What must the tension in the rope be for equilibrium?



As usual, this is a "torque about the pin" problem with the angular acceleration equal to zero. Using "r-perpendicular" on the "mg" term, and "F-perpendicular" on the tension force, we can write:

$$\sum \Gamma_{pin} :$$

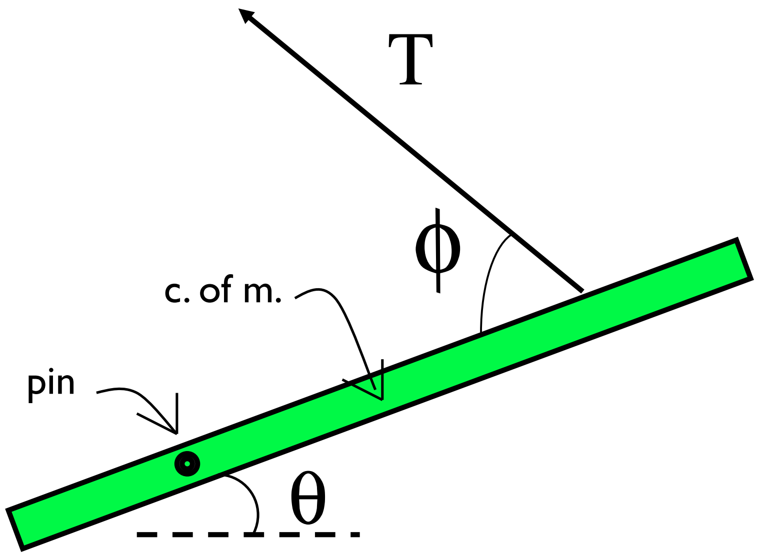
$$\cancelto{0}{\Gamma_H} + \cancelto{0}{\Gamma_V} - mg \quad (r_\perp) \quad + \quad (T_\perp) \left(\frac{L}{2}\right) = I_{pin} \cancelto{0}{\alpha}$$

$$-mg\left(\frac{\cancel{L}}{4}\cos\theta\right) + (T\sin\phi)\left(\frac{\cancel{L}}{2}\right) = I_{pin}\alpha$$

$$\Rightarrow \quad T = \frac{mg(\cos\theta)}{2\sin\phi}$$

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

c.) Use the *Parallel Axis Theorem* to derive an expression for the beam's *moment of inertia* about the pin.



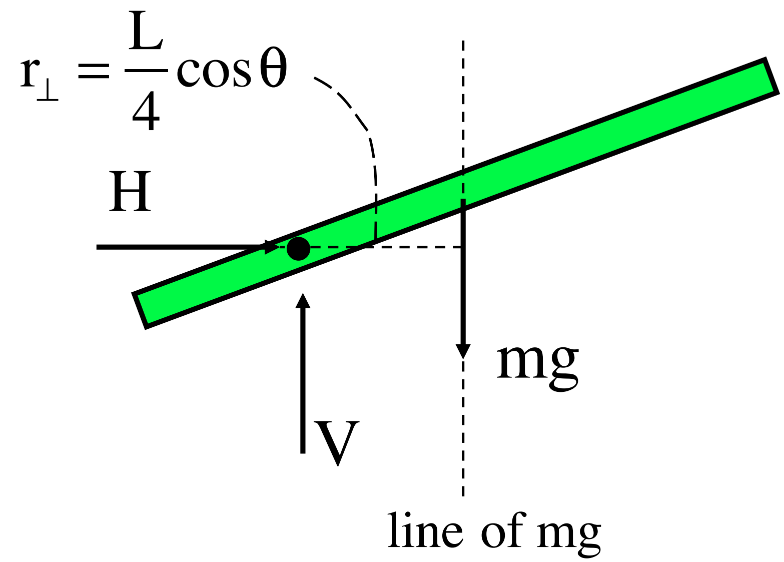
$$I_p = I_{cm} + md^2$$

$$= \frac{1}{12} mL^2 + m\left(\frac{L}{4}\right)^2$$

$$= \frac{7}{48} mL^2$$

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

d.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial *angular acceleration* about the pin?



$$\sum \Gamma_{pin} \doteq$$

$$\Gamma_H^{\,0} + \Gamma_V^{\,0} - mg\,(r_\perp) = -I_{pin}\alpha$$

$$\Rightarrow \quad -mg\left(\frac{L}{4}\cos\theta\right) = -I_{pin}\alpha$$

$$\Rightarrow \quad \alpha = \frac{mg\left(\frac{L}{4}\cos\theta\right)}{\frac{7}{48}mL^2}$$

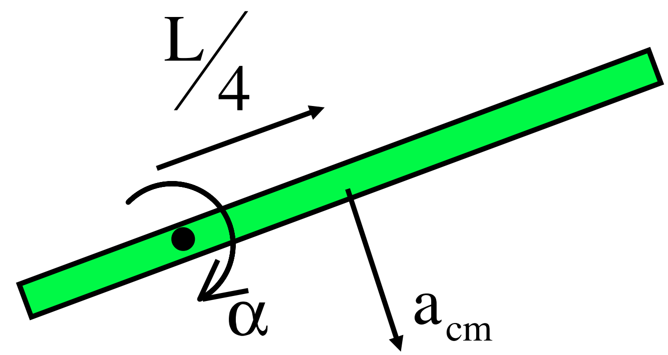
$$\Rightarrow \quad \alpha = \frac{12g(\cos\theta)}{7L}$$

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

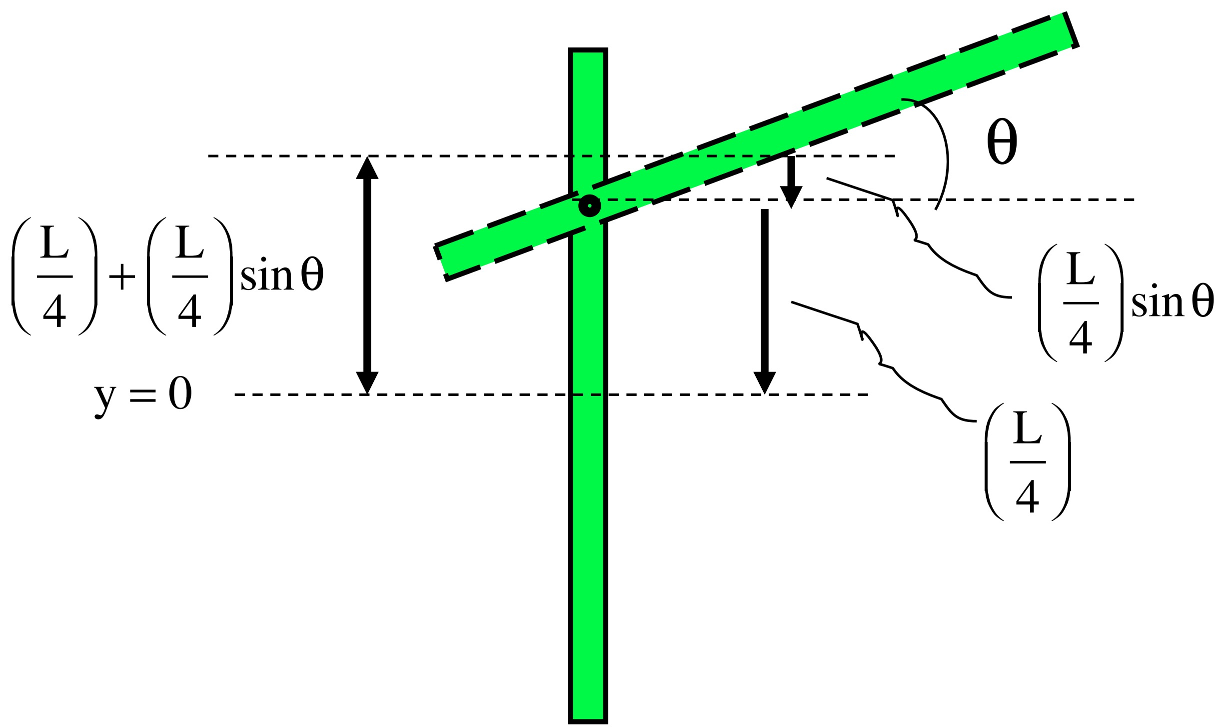
e.) What is the initial *acceleration* of the beam's center of mass?

$$a_{cm} = r_{cm}\alpha$$

$$= \left(\frac{L}{4}\right)\alpha$$



f.) The beam rotates downward. What is its angular velocity as it passes through the vertical?



$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

f.) (con' t.) (See sketch on the previous page.)

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mg\left(\frac{L}{4} + \frac{L}{4}\sin\theta\right) + 0 = \left(\frac{1}{2} I_{pin}\omega^2\right) + 0$$

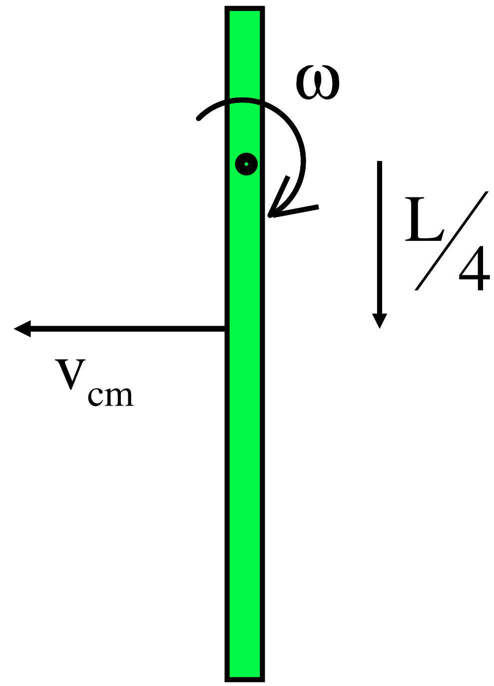
$$0 + \cancel{m}g\left(\frac{\cancel{L}}{4} + \frac{\cancel{L}}{4}\sin\theta\right) + 0 = \left(\frac{1}{2}\left(\frac{7}{48}\cancel{m}L^{\cancel{2}}\right)\omega^2\right) + 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{24}{7L} g(1 + \sin\theta)}$$

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

g.) What is the beam's center-of-mass velocity at that point?

Knowing ω, you can easily determine v.



$$v_{cm} = r_{cm}\omega$$
$$= \left(\frac{L}{4}\right)\omega$$

h.) What is the beam's *angular momentum* "L" about the pin at that point?

$$L = I_{pin}\omega$$
$$= \left(\frac{7}{48} mL^2\right)\omega$$